Table 1 Effects of varying K

K	∞	6.8	1.4	1.0	0.9	0.8	0.7	0.6	0.55
ψ_s	0.136	0.136	0.139	0.143	0.145	0.148	0.153	0.165	0.185
θ_s	0.0	0.020	0.100	0.143	0.161	0.184	0.218	0.274	0.336
K	0.55	0.50	0.45	0.40					
$(\psi\eta_1^2)_{max}$	3.02	2.63	2.29	1.98					

where

$$u = v = w = 0,\ y = 0$$

$$u \to r\Omega(1 - K\theta),\ w \to 0,\ y \to \infty \tag{4}$$

Put $y = (2\theta\nu/\Omega)^{1/2}\eta$, and $\gamma_1 = \eta/\eta_1$, $\gamma_2 = \eta/\eta_2$. Assume that approximately

$$u = r\Omega(1 - K\theta)\left[\frac{3\gamma_1}{2} - \frac{\gamma_1^3}{2} - \frac{K\theta\eta_1^2}{2}\gamma_1(1-\gamma_1)^2\right],\quad 0 \leqslant \gamma_1 \leqslant 1$$

$$= r\Omega(1 - K\theta),\ \gamma_1 \geqslant 1 \tag{5}$$

and

$$w = (r\Omega\theta/2)\eta_2^2(1 - K^2\theta^2)(\gamma_2 - 2\gamma_2^2 + \gamma_2^3),\ 0 \leqslant \gamma_2 \leqslant 1$$

$$= 0,\ \gamma_2 \geqslant 1 \tag{6}$$

These expressions satisfy Eqs. (1) and (2) at the surface $y = 0$, together with the boundary conditions in Eq. (4). If they are substituted into the equations obtained by integrating (1) and (2) with respect to y across the boundary layer [making use of Eq. (3)], the resulting equations are of the following form:

$$d\eta_1/d\psi = F(\psi, \eta_1) + (1/K^2)G(\psi, \eta_1, \eta_2) \tag{7}$$

and

$$d\eta_2/d\psi = H(\psi, \eta_1, \eta_2)(d\eta_1/d\psi) + I(\psi, \eta_1, \eta_2) + (1/K^2)L(\psi, \eta_1, \eta_2) \tag{8}$$

where $\psi = K\theta$. These equations have been integrated concurrently for various values of K by a step-by-step process, using the Ace computer of the Mathematics Division of the National Physical Laboratory. If the solution indicates that $\psi\eta_1^2$ reaches the value 3, then at this point Eq. (5) shows that $\partial u/\partial y$ is zero at the surface, implying separation of the u component profile. If K is very large and ψ is finite at separation, this means that θ at separation must be very small, and the rotation can have had no appreciable effect on the flow. Thus when $K \to \infty$, so that the terms in G and L of Eqs. (7) and (8) vanish, the former equation becomes equivalent to that for a two-dimensional flow with a linear adverse external-velocity gradient, as solved by Howarth.[3] The accurate numerical solution of this problem has $\psi = 0.120$ at separation. Using step lengths in ψ of 0.01 and 0.005, one obtains $\psi = 0.136$ and 0.134, respectively, at separation. The errors due to the approximations made in Eqs. (5) and (6) are thus fairly small. It was considered sufficiently accurate to use the step length $\psi = 0.01$ in the subsequent calculations for smaller values of K. Here separation does not occur so close to the leading edge, and the rotation has an effect on the flow, postponing separation till a higher value of ψ is reached. Thus the pressure rise between the leading edge and separation is increased. This is shown in Table 1, where ψ_s is the value of ψ at separation, θ_s being the corresponding angle in radians.

For values of K less than about 0.548, $\psi\eta_1^2$, which near the leading edge increases with increasing ψ, reaches a maximum value of less than 3 and then decreases again. Thus the separation condition is never reached, and presumably the boundary layer is stabilized completely against separation by a linear adverse external-velocity gradient.

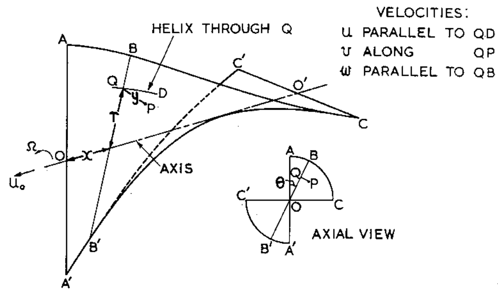

Fig. 1 Helical surface

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A Further Note on Propagation of Thermal Disturbances in Rarefied-Gas Flows

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In a recent note,[1] the small-disturbance rarefied-gas equations for one-dimensional nonsteady flow were shown to satisfy the characteristic equations

$$\left\{\frac{\partial}{\partial t} \pm 0.813\frac{\partial}{\partial x}\right\}P_{1\pm} = \left(0.487\frac{H}{p_0} \mp 0.417\frac{F}{c_0}\right)\frac{L}{c_0} + \frac{L}{t_f c_0}\left(0.15\frac{\tau}{p_0} \mp 0.4\frac{q}{p_0 c_0}\right) \tag{1}$$

$$\left\{\frac{\partial}{\partial t} \pm 2.13\frac{\partial}{\partial x}\right\}P_{2\pm} = \left(1.78\frac{H}{p_0} \pm 1.66\frac{F}{c_0}\right)\frac{L}{c_0} - \frac{L}{t_f c_0}\left(\frac{1.57\tau}{p_0} \pm 0.4\frac{q}{p_0 c_0}\right) \tag{2}$$

assuming the existence of external heat addition $H(x,t)$ and external forces $F(x,t)$ and including changes in the characteristic quantities as $t \to t_f$. The characteristic quantities $P_{1\pm}$ and $P_{2\pm}$ are defined by

$$P_{1\pm} = [(\theta - 0.51p - 0.11(\tau/p_0)] \pm (1/c_0)[0.33(q/p_0) - 0.42u] \tag{3}$$

$$P_{2\pm} = [\theta + 0.78p + 1.18(\tau/p_0)] \pm (1/c_0)[0.85(q/p_0) + 1.66u] \tag{4}$$

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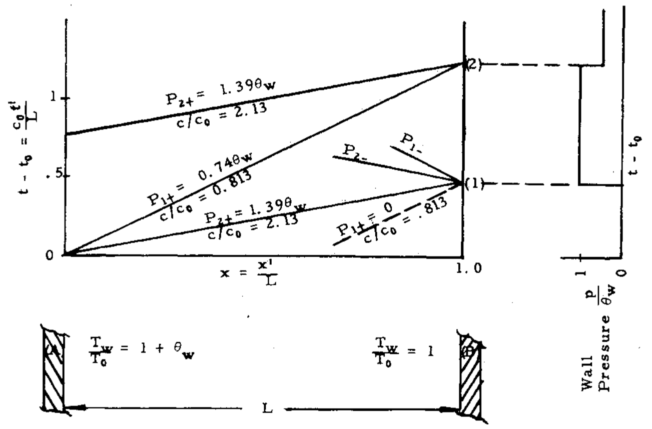

Fig. 1 Thermal disturbance propagation in very rarefied-gas field

Here p is the perturbation pressure, θ the perturbation temperature, u the velocity, τ the normal stress, and q the heat flux. The relaxation time is defined by $t_f = \mu_0/p_0$. In Eqs. (1) and (2), x,t, and the propagation velocities c_1 and c_2 are written in the dimensionless form

$$\frac{L}{x'} = \frac{1}{x} \qquad \frac{L}{c_0 t'} = \frac{1}{t} \qquad \frac{c_1}{c_0} = 0.813 \qquad \frac{c_2}{c_0} = 2.13$$

As an illustrative example, the disturbance produced by a plate suddenly heated in a rarefied-gas field, initially in equilibrium at a temperature T_0, can be calculated assuming $F = H = 0$, $L/c_0 \ll t_f$ and that specular reflection does not occur at the plate. The field particles are absorbed and re-emitted with a Maxwellian distribution at the wall temperature T_w. Continuity of mass and the equation of state require that[2]

$$p = 0 = s_w + (\theta_w/2) \qquad p_w = \theta_w/2 \qquad u = 0$$

The characteristic values at the heated wall are $P_{2+} = 1.39\,\theta_w$, $P_{1+} = 0.74\theta_w$. At the unheated wall (1), $\theta = 0$, and the fast characteristic results in an average pressure $p = 1.08\theta_w$. At wall (2), the slow and fast characteristics yield an average pressure $p = 0.5\theta_w$ (Fig. 1).

The heated plate consequently produces a pressure disturbance that is transmitted by the rarefied-gas field and results in a positive pressure or repulsive force at (B) when $\theta_w > 0$ and a negative pressure or attractive force when $\theta_w < 0$.

This thermal disturbance propagation in the rarefied-gas field which occurs in the limit $L/t_f c_0 \ll 1$, $F = H = 0$ is unique in that relatively few field particle collisions occur during the propagation. Disturbances originated at a boundary will not be altered during propagation by other disturbances existing in the field and will be altered only by collisions at another boundary. Therefore, the thermal disturbances initiated at boundaries will propagate unchanged in the field.

In the limit $t_f \gg t$, when $F = H = 0$, the one-dimensional equations for longitudinal disturbances can be written in the form

$$\frac{\partial}{\partial t}\left(\theta - 0.51p - 0.11\frac{\tau}{p_0}\right) + 0.813\,\frac{\partial}{\partial x}\left(0.33\,\frac{q}{p_0 c_0} - 0.42\,\frac{u}{c_0}\right) = 0 \quad (5)$$

$$\frac{\partial}{\partial t}\left(0.33\,\frac{q}{p_0 c_0} - 0.42\,\frac{u}{c_0}\right) + 0.813\,\frac{\partial}{\partial x}\left(\theta - 0.51p - 0.11\,\frac{\tau}{p_0}\right) = 0 \quad (6)$$

$$\frac{\partial}{\partial t}\left(\theta + 0.78p + 1.18\,\frac{\tau}{p_0}\right) + 2.13\,\frac{\partial}{\partial x}\left(0.85\,\frac{q}{p_0 c_0} + 1.66\,\frac{u}{c_0}\right) = 0 \quad (7)$$

$$\frac{\partial}{\partial t}\left(0.85\,\frac{q}{p_0 c_0} + 1.66\,\frac{u}{c_0}\right) + 2.13\,\frac{\partial}{\partial x}\left(\theta + 0.78p + 1.18\,\frac{\tau}{p_0}\right) = 0 \quad (8)$$

By elimination of the terms containing u and q, the following propagation equations are obtained for the longitudinal temperature, stress, and pressure disturbances:

$$\frac{\partial^2}{\partial t^2}\left(\theta - 0.51p - 0.11\,\frac{\tau}{p_0}\right) = (0.813)^2\,\frac{\partial^2}{\partial x^2}\left(\theta - 0.51p - 0.11\,\frac{\tau}{p_0}\right) \quad (9)$$

$$\frac{\partial^2}{\partial t^2}\left(\theta + 0.78p + 1.18\,\frac{\tau}{p_0}\right) = (2.13)^2\,\frac{\partial^2}{\partial x^2}\left(\theta + 0.78p + 1.18\,\frac{\tau}{p_0}\right) \quad (10)$$

The equations for the propagation of small plane disturbances, Eqs. (1) and (2), also may be written in the following form:

$$[\mathbf{n}(1/c)(\partial/\partial t) \pm \nabla]\cdot\mathbf{P}_{1,2\pm} = 0$$

$$[\mathbf{n}(1/c)(\partial/\partial t) \pm \nabla]\ \mathbf{x}\ \mathbf{P}_{1,2\pm} = 0$$

where, $t_f \gg t$, $F = H = 0$, $\mathbf{P}_{1,2} = \mathbf{i}P_{1,2}$, c is the dimensionless propagation velocity, and $\mathbf{n}$ is a unit vector along the direction of propagation. The forward propagating plane longitudinal disturbances consequently satisfy equations of the form

$$\nabla\cdot\mathbf{P} + (1/c)(\partial\mathbf{n}\cdot\mathbf{P})/\partial t] = 0 \quad (11)$$

$$\nabla\ \mathbf{x}\ \mathbf{P} = 0 \quad (12)$$

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A Further Note on Propagation of Transverse Disturbances in Rarefied-Gas Flows

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Lees and Yang[1] recently have shown that the two-dimensional Grad equations for the rarefied-gas field, when applied to the Rayleigh problem, indicate the propagation of small transverse shear disturbances along distinct char-

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